Vectors and Matrices

Unit 1 · Linear Systems and Matrix Algebra

Concept 1.1 · Interactive Reading Guide

Formulating Systems from Real-World Scenarios

These notes focus on translation only: turning a contextual narrative—such as a pricing model, a resource constraint, a mixture, or an allocation problem—into a system of equations with every variable and constant clearly identified. In this unit, each scenario produces a linear system; solving begins in Concept 1.2.

Learning Objective

Construct a system of linear equations from a contextual narrative, explicitly defining all variables and constant values.

Key Definitions

System of equations.
Write the definition.

Linear equation.
Write the definition.

Standard form of a linear system.
Write the definition.

1.1.1 Example: Pricing Model

> Rowan is repairing their cargo van and purchases 4 spark plugs and 2 quarts of synthetic oil for $34. The next day, Quinn works on a different vehicle and purchases 6 spark plugs and 5 quarts of the same synthetic oil for $73.

Define the unknowns.

Workspace: Define the unknowns.

Translate the conditions.

Workspace: Translate the conditions.

1.1.2 Example: Resource Constraint

> Each poster uses 3 square feet of vinyl and 2 hours of labor. Each banner uses 6 square feet of vinyl and 3 hours of labor. An order uses exactly 24 square feet of vinyl and 14 hours of labor.

Define the unknowns.

Workspace: Define the unknowns.

Translate the conditions.

Workspace: Translate the conditions.

1.1.3 Example: Mixture

> Colombian coffee costs $8 per pound, and Sumatran coffee costs $14 per pound. The café wants 30 pounds of a blend worth $12 per pound.

Define the unknowns.

Workspace: Define the unknowns.

Translate the conditions.

Workspace: Translate the conditions.

1.1.4 Example: Three-Variable Ticket Allocation

> A community theater sells student tickets for $8, adult tickets for $15, and senior tickets for $10. For Friday's performance, the theater sells 100 tickets in total and collects $1,190 in ticket revenue. Student and senior tickets qualify as discounted admissions, and exactly 50 discounted tickets are sold.

Define the unknowns.

Workspace: Define the unknowns.

Translate the conditions.

Workspace: Translate the conditions.

1.1.5 The Formulation Habit

1. Define the unknowns.
   – Assign one variable to each unknown quantity the scenario asks for, and state its unit. A statement such as "Let p equal the price of one spark plug, in dollars" is complete; "Let p equal spark plugs" is not.
2. Separate the conditions.
   – Identify each independent constraint, relationship, or total in the narrative. Each independent condition becomes one equation.
3. Translate and balance units.
   – Express how the known constants and unknown variables interact, ensuring both sides of each equation measure the exact same kind of quantity (dollars equal dollars; hours equal hours). Counts, rates, and totals given in the scenario act as constants in the model, not extra variables.
4. Assemble the system.
   – Group the equations together in a consistent, readable structure so they hold simultaneously. Throughout this unit, our scenarios yield linear systems written in standard form.

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1.1.6 Translation Pitfalls

- Multiply unit rates by quantities to express total contributions.
  - When a scenario gives a rate per unit—dollars per spark plug in Example 1.1.1, square feet per poster in Example 1.1.2, or dollars per ticket in Example 1.1.4—multiply that rate by the corresponding quantity so the term represents the total contribution (for example, $4p$ dollars for 4 spark plugs in Example 1.1.1).
- Keep each constant paired with its intended variable.
  - In Rowan's purchase in Example 1.1.1 ($4p + 2q = 34$), 4 counts the spark plugs and 2 counts the quarts of oil; swapping parameters alters the constraint.
- Match units across the equals sign.
  - Both sides of an equation must measure the exact same kind of quantity. In Example 1.1.2, square feet of vinyl equal square feet of vinyl; in Examples 1.1.1, 1.1.3, and 1.1.4, dollars equal dollars.
- Convert target rates into totals before equating values.
  - In the coffee mixture of Example 1.1.3, \$12 is a price per pound, not the total worth of the batch. Multiply by the 30-pound weight first ($12 \cdot 30 = 360$) so both sides of the value equation measure total dollars.
- Preserve variable order and note missing terms.
  - Keeping variables in the same left-to-right order in every equation, and recognizing a missing variable as having coefficient 0, prevents alignment mistakes when translating systems into matrices later in Unit 1. Example 1.1.4 writes the discounted-ticket condition as $x + 0y + z = 50$.

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Alkimi Sandbox 1: Verify Your Work

A landscaper purchases 8 maple trees and 12 oak trees for $1,440. The next week, the landscaper purchases 5 maple trees and 9 oak trees for $990.

Define the two variables and write the system of equations that models this scenario. Check your work in Alkimi Sandbox 1 on the Canvas page.

Define the unknowns.

Workspace: Define the unknowns.

Translate the conditions.

Workspace: Translate the conditions.